Department of Mathematics

PLP: An Introduction to Mathematical Proof

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What is PLP?

A set of resources for an introductory course to proof writing in mathematics. These resources are currently being used to teach MATH 220 at UBC. Resources include:

- Textbook
- Video and accompanying slides that introduce each core course topic
- Worksheets

Available at: https://personal.math.ubc.ca/~PLP/

Why was this project initiated?

Standardization: the content of the course used to change from year to year depending on the Instructor in Charge. This book and the corresponding auxiliary material made it easily adaptable from year to year, setting a standard for the course.

Accessibility: all course materials now available for students online for free

Active learning: auxiliary materials, including videos and worksheets, can be used by instructors to smoothly implement active learning in the classroom

Restructured curriculum and presentation of materials to address common challenges students face when learning how to write proofs.

- Proofs are introduced **as early as possible**
- Proof writing questions are accompanied with **hints, scratch work,** and **formal solutions**

Example from textbook

Problem: Asks students to prove a math statement is true

Let x, y be positive real numbers. Without using Calculus, prove that

$$(x > y) \implies (\sqrt{x} > \sqrt{y})$$

Hint: A helpful observation, tip, or suggested proof technique

Try writing the inequality in a different way, the difference of squares might help.

Scratch work: A look behind the curtain on how you could approach the problem and why, and discussion on potential misconceptions

We see that what we want to prove in this question is a conditional statement. So, we can assume the hypothesis and try to show the conclusion. So, we assume $x > y$ and try to show that $\sqrt{x} > \sqrt{y}$. Since we cannot simply take the square roots of both sides (because of the above argument), we need to find a different way to prove the conclusion.

We see that we can rewrite the hypothesis as $x - y > 0$. We also know that

$$\underbrace{x - y}_{>0} = (\sqrt{x})^2 - \sqrt{y})^2 = (\sqrt{x} - \sqrt{y})\underbrace{(\sqrt{x} + \sqrt{y})}_{>0},$$

where we have factored the difference of squares: $a^2 - b^2 = (a - b)(a + b)$. This tells us that $\sqrt{x} - \sqrt{y} > 0$, which is what we wanted to show. Now, we can write this nicely in a proof.

Proof: A formal solution using scratch work

Assume that $0 < y < x$. This means that $x - y > 0$. Then, since $x, y > 0$, we can factor the expression $x - y$ and get

$$(\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y}) > 0$$

and since the square root function is nonnegative, and $x, y > 0$, we also know that $\sqrt{y} + \sqrt{x} > 0$. So dividing both sides of the inequality by $\sqrt{x} + \sqrt{y}$ gives

$$\sqrt{x} - \sqrt{y} > 0$$

Hence we conclude that $\sqrt{x} > \sqrt{y}$ as required.

Experience in MATH 220

- Students watch videos before class: introduce topic, definitions, and a few examples
- Students take online Canvas quiz on video content
- Class time is devoted to examples and active learning
- Worksheets make active learning easy to implement: list videos to watch before and after completing

Student Comments (from SEI)

- "The pre–recorded videos by Andrew were VERY helpful to my learning, and saved lots of lecture time."
- "Classes are very interactive, and I actually enjoy attending his classes, and I look forward to them."
- *On strengths of the course:* "Used pre-lecture videos, encouraged students to try problems before going over them in class"
- *On strengths of the course:* "Active learning through practicing examples and comPAIR where we can see the progress of our peers and compare, video lectures"

Further work

Student As Partners: Previous MATH 220 students have written problems featuring scratch work and hints for future years; gain student perspective on common misconceptions and problems to prioritize

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